MULTI-FOCUS IMAGE FUSION VIA COUPLED DICTIONARY TRAINING

Rui Gao[† ‡], Sergiy A. Vorobyov[‡], and Hong Zhao[†]

[†]Dept. Computer application technology, Northeastern University, Shenyang 110819, China
[‡]Dept. Signal Processing and Acoustics, Aalto University, P.O. Box 13000, FI-00076 Aalto, Finland
Emails: gaorui@research.neu.edu.cn, svor@ieee.org, zhaoh@neusoft.com

ABSTRACT

A novel multi-focus image fusion approach using coupled dictionary training is proposed. It exploits the facts that (i) the patches in example data can be sparsely represented by a couple of over-complete dictionaries related to the focused and blurred categories of images and (ii) merging such representations is better than just selecting the sparsest one in the estimate of the original image. Inspired by these observations, we enforce the similarity of sparse representations between the focused and blurred image patches by jointly training the coupled dictionary, and then fuse these representations to generate an all-in-focus image by a fusion rule. The key characteristics of our approach are bridging the gap between coupled dictionaries, combining plain averaging and "choose-max" as an appropriate fusion rule, and forming a more accurate representation, compared to existing approaches which simply admit sparse representation over one dictionary. Extensive experimental comparisons with state-of-the-art multi-focus image fusion algorithms validate the effectiveness of the proposed approach.

Index Terms— Image fusion, sparse representations, coupled dictionary training, K-SVD, multi-focus image

1. INTRODUCTION

Because of the limited depth of field in optical lenses of conventional cameras, it is impossible to capture an all-in-focus image without sacrificing image quality or using specialized optic sensors [1]. Multi-focus image fusion is an effective technique to solve this problem by fusing multiple images captured with different focus distances [2]–[4]. Differences in employing multi-focus image fusion appear mainly due to the uses of different fusion domains. The majority of literature is traditionally categorized into two basic approaches: the *spatial domain-based* approach and the *transform domain-based* approach. In the first category, the methods such as image fusion based on Laplacian pyramid (LP) [5], spatial frequency (SF) [6], multi-scale weighted gradient (MWG) [7], and variance [8] directly select the best regions to fuse multiple images. The second category suggests to apply multiscale transforms to decompose source images and construct an all-in-focus image in the inverse transform domain. Algorithms of this type include discrete wavelet transform (DWT) [9], curvelet transform (CVT) [2], non-subsampled contourlet transform (NSCT) [10] among others. These methods are still sensitive to image misregistration and they may suffer from undesirable artifacts. Recently, sparsity and overcompleteness have been successfully used for image fusion [11]–[16]. These approaches exploit the fact that natural images can be compactly expressed over an overcomplete dictionary as a linear combination of sparse coefficients. The coefficients are combined suitably to reconstruct an all-in-focus image by the fusion rule. The success of such approach depends on how accurately it describes the images and how efficiently it fuses these coefficients.

In this paper, we propose a novel multi-focus image fusion scheme based on a coupled dictionary. Given pairs of training images, we seek two dictionaries that lead to the best possible representation for the focused and blurred categories of images. Using the two dictionaries, we then introduce the "averaging" fusion rule to find accurate sparse representation for reconstructing an all-in-focus image. We aim at improving the performance of fusion method. The main difference of the proposed approach from the existing approaches is that the coupled dictionary is exploited for more compact and accurate representation of different focus images. To improve the accuracy and efficiency of the proposed approach, new fusion rule is defined and improved K-SVD-based joint coupled dictionary training algorithm is proposed.

2. PROPOSED METHOD

Accurate sparse representation is crucial to successful image fusion. The classical model uses one overcomplete dictionary to describe source images which contain the focused and blurred categories of features. The properties of such dictionary set the limits on the sparsity of the coefficients. This paper has at the disposal two pieces of knowledge: the first one is sparse representations by a coupled dictionary, and the other is that merging multiple sparse coefficients on one signal is better than just selecting the sparsest one alone. With these, it is suggested to jointly learn the coupled dictionary from the focused and blurred image patches, and then

use these dictionaries to describe original images in terms of sparse coefficients. Merging these coefficients forces to produce a better estimated representation by fusion rule. As such, this contribution differs from previous contributions on the basis of sparse model used. Such approach can outperform existing approaches which simply use sparse representation over one dictionary.

2.1. Coupled Dictionary Training

Given the sampled training vector pair $P \triangleq \left\{ \boldsymbol{X}^F, \boldsymbol{X}^B \right\}$, we define $\boldsymbol{X}^F \triangleq \left[\boldsymbol{x}_1^F, \boldsymbol{x}_2^F, \cdots, \boldsymbol{x}_n^F \right] \in \mathbb{R}^{d \times n}$ as the matrix of n sampled focused image vectors and $\boldsymbol{X}^B \triangleq \left[\boldsymbol{x}_1^B, \boldsymbol{x}_2^B, \cdots, \boldsymbol{x}_n^B \right] \in \mathbb{R}^{d \times n}$ as the matrix of corresponding blurred image vectors which are created by $\boldsymbol{X}^F$ using Gaussian blur function. Here d is the dimension of the sampled image vectors. Motivated by [17] and [18], we intend to find two dictionaries $\boldsymbol{D}^F$, $\boldsymbol{D}^B \in \mathbb{R}^{d \times N}$ with respect to the same sparse representation for the couple of feature spaces. The classical coupled dictionary training problem is expressed as

$$\min_{\boldsymbol{D}^F, \boldsymbol{D}^B, \boldsymbol{\Gamma}} \left\| \boldsymbol{X}^F - \boldsymbol{D}^F \boldsymbol{\Gamma} \right\|_2^2 + \left\| \boldsymbol{X}^B - \boldsymbol{D}^B \boldsymbol{\Gamma} \right\|_2^2$$

$$\text{s.t.} \left\| \boldsymbol{\Gamma} \right\|_0 \leqslant T_0, \left\| \boldsymbol{d}_i^F \right\|_2^2 \leqslant 1, \left\| \boldsymbol{d}_i^B \right\|_2^2 \leqslant 1, \forall 1 \leqslant i \leqslant N \quad (1)$$

where $\boldsymbol{\Gamma}$ is the joint spare coding of $\boldsymbol{X}^F$ and $\boldsymbol{X}^B$, $\boldsymbol{d}_i^F$ and $\boldsymbol{d}_i^B$ are the i-th columns of $\boldsymbol{D}^F$ and $\boldsymbol{D}^B$, respectively, and T_0 is the parameter controlling the sparsity penalty. Many approaches have been developed for learning strategy [18]–[22]. They separate the objective function (1) into two subproblems, namely sparse coding and dictionary updating. The coefficients are estimated via l_1-norm minimization, keeping the two dictionaries fixed alternately, and the dictionaries are estimated through least squares, keeping the coefficients fixed. These approaches differ in the estimation of sparse coefficients and the update rules of coupled dictionary. However, the reliability and complexity of learning strategy still need to be improved.

The K-SVD as an iterative algorithm accelerates the convergence by the alternating update of the dictionary atoms and the corresponding coefficients in the dictionary updating stage [23],[24]. Here, we present the coupled dictionary training procedure using improving K-SVD algorithm. The main innovation is to keep the supports in $\boldsymbol{\Gamma}$ intact, seeking the updates of $\boldsymbol{D}^F$, $\boldsymbol{D}^B$, and $\boldsymbol{\Gamma}$, alternately. The corresponding training optimization problem is then given as

$$\min_{\boldsymbol{D}^F, \boldsymbol{D}^B, \boldsymbol{\Gamma}} \left\| \boldsymbol{X}^F - \boldsymbol{D}^F \boldsymbol{\Gamma} \right\|_2^2 + \left\| \boldsymbol{X}^B - \boldsymbol{D}^B \boldsymbol{\Gamma} \right\|_2^2$$

$$\text{s.t.} \quad \left\| \boldsymbol{\Gamma} \right\|_0 \leqslant T_0, \quad \boldsymbol{\Gamma} \odot \mathbf{M} = 0 \quad (2)$$

where $\odot$ represents element-wise product and the mask matrix $\mathbf{M}$ keeps all the zero atoms in $\boldsymbol{\Gamma}$ intact. Define $\left\{ \boldsymbol{\gamma}_i^T \right\}_{i=1}^N$ and $\left\{ \mathbf{m}_i^T \right\}_{i=1}^N$ as the rows of $\boldsymbol{\Gamma}$ and $\mathbf{M}$, respectively. Here $\mathbf{m}_i^T = \left\{ \left| \boldsymbol{\gamma}_i^T \right| = 0 \right\}$. For optimizing $\boldsymbol{d}_i^F$, $\boldsymbol{d}_i^B$, and $\boldsymbol{\gamma}_i$ in the

dictionary updating stage, we exploit the separability of the objective function in (2) for $\boldsymbol{D}^F$ and $\boldsymbol{D}^F$, and decompose it into the following two problems

$$\left\{ \underline{\boldsymbol{D}}^F, \underline{\boldsymbol{\Gamma}} \right\} = \arg \min_{\boldsymbol{D}^F, \boldsymbol{\Gamma}} \left\| \boldsymbol{X}^F - \sum_{i=1}^N \boldsymbol{d}_i^F \boldsymbol{\gamma}_i^T \right\|_2^2 \quad (3)$$

$$\left\{ \underline{\boldsymbol{D}}^B, \underline{\boldsymbol{\Gamma}} \right\} = \arg \min_{\boldsymbol{D}^B, \boldsymbol{\Gamma}} \left\| \boldsymbol{X}^B - \sum_{i=1}^N \boldsymbol{d}_i^B \boldsymbol{\gamma}_i^T \right\|_2^2 \quad (4)$$

where we introduce the error matrices without the j-th atom, and directly apply a singular value decomposition (SVD) operation to these matrices. The error matrices are defined as

$$\boldsymbol{E}_j^F \triangleq \left(\boldsymbol{X}^F - \sum_{i \neq j} \boldsymbol{d}_i^F \boldsymbol{\gamma}_i^T \right) \odot \left(\mathbf{1}_d \cdot \mathbf{m}_i^T \right) \quad (5)$$

$$\boldsymbol{E}_j^B \triangleq \left(\boldsymbol{X}^B - \sum_{i \neq j} \boldsymbol{d}_i^B \boldsymbol{\gamma}_i^T \right) \odot \left(\mathbf{1}_d \cdot \mathbf{m}_i^T \right) \quad (6)$$

where $\mathbf{1}_d$ represents the d time replications of $\mathbf{m}_i^T$. Such approach allows to effectively update all the atoms corresponding to $\boldsymbol{X}^F$ and $\boldsymbol{X}^B$, without using rank-1 approximation which performs one atom at a time.

In the sparse coding stage, we search the sparsest matrix $\boldsymbol{\Gamma}$ for the signals $\boldsymbol{X}^F$ and $\boldsymbol{X}^B$. The sparse representation problems are then given as

$$\underline{\boldsymbol{\Gamma}} = \arg\min_{\boldsymbol{\Gamma}} \left\| \boldsymbol{X}^F - \boldsymbol{D}^F \boldsymbol{\Gamma} \right\|_2^2 \quad \text{s.t.} \left\| \boldsymbol{\Gamma} \right\|_0 \leqslant T_0 \quad (7)$$

$$\underline{\boldsymbol{\Gamma}} = \arg\min_{\boldsymbol{\Gamma}} \left\| \boldsymbol{X}^B - \boldsymbol{D}^B \boldsymbol{\Gamma} \right\|_2^2 \quad \text{s.t.} \left\| \boldsymbol{\Gamma} \right\|_0 \leqslant T_0. \quad (8)$$

The problems (7) and (8) are solved by the coefficient reuse-orthogonal matching pursuit (CoefROMP) [24], using two dictionaries alternately.

2.2. Fusion Rule

Let the source images $\boldsymbol{I}_1, \boldsymbol{I}_2, \cdots, \boldsymbol{I}_n$ be acquired with different focus parameters from the same scene. Our goal is to define the fusion rule to merge these images into the all-in-focus image. Using one dictionary, as in the existing approach, implies that the focused and blurred features in each image are represented by one linear combination. Thus, accurate representation for blurred features is impossible. Instead, we propose to use a collection of sparse representations, with respect to the focused and blurred categories of dictionaries, in order to produce a better sparse representation. For each input image $\boldsymbol{I}_k$, we generate two sparse representations with respect to the couple of dictionaries $\left\{ \boldsymbol{D}^F, \boldsymbol{D}^B \right\}$. With the aim to fuse these images, the sparse model suggests to seek accurate representations of the salient features. The problems of generating the two sparsest representations of $\boldsymbol{I}_k$ can be formulated as

$$\min \left\| \boldsymbol{A}_k^F \right\|_1 \quad \text{s.t.} \quad \left\| \boldsymbol{D}^F \boldsymbol{A}_k^F - \boldsymbol{I}_k \right\|_2^2 \leq \epsilon \quad (9)$$

$$\min \left\| \boldsymbol{A}_k^B \right\|_1 \quad \text{s.t.} \quad \left\| \boldsymbol{D}^B \boldsymbol{A}_k^B - \boldsymbol{I}_k \right\|_2^2 \leq \epsilon \quad (10)$$

where ϵ is the tolerance error parameter, and $\boldsymbol{A}_k^F$ and $\boldsymbol{A}_k^B$ are the matrices of sparse coefficients over the dictionaries $\boldsymbol{D}^F$ and $\boldsymbol{D}^B$, respectively.

Based on the fact that each set of sparse coefficients represents its own salient features, we can conclude that averaging these coefficients leads to the fusion of different features. For the j-th column vectors $\left[\boldsymbol{A}_k^F\right]_j$ in $\boldsymbol{A}_k^F$ and $\left[\boldsymbol{A}_k^B\right]_j$ in $\boldsymbol{A}_k^B$, $j = 1, 2, \cdots, N$, we compute

$$\left[\boldsymbol{A}_k^P\right]_j = \left(\left[\boldsymbol{A}_k^F\right]_j + \left[\boldsymbol{A}_k^B\right]_j\right)/2 \qquad (11)$$

where $\left[\boldsymbol{A}_k^P\right]_j$ is the j-th vector of averaging coefficients. After obtaining the sparse representations for each image, the sliding window technique is used to divide the sparse vectors $\left[\boldsymbol{A}_k^P\right]_j$ into M small blocks $\left[\boldsymbol{A}_k^P\right]_j(m)$, $m = 1, 2, \cdots, M$, from left-top to right-bottom. For each pair of corresponding blocks, we then calculate the value

$$T_{k,j}(m) = \left\|\left[\boldsymbol{A}_k^P\right]_j(m)\right\|_1. \qquad (12)$$

As a general rule, the block with a bigger value is chosen to construct the fused image. We directly use the following "choose-max" rule

$$\boldsymbol{\alpha}_j(m) = \left[\boldsymbol{A}_{\underline{k}}^P\right]_{\underline{j}}(m), \ \{\underline{k}, \underline{j}\} = \arg\max_{k,j}\left(T_{k,j}(m)\right). \qquad (13)$$

Here $\boldsymbol{\alpha}_j$ is the vector of the composite coefficients $\boldsymbol{A}$, and $\left[\boldsymbol{A}_{\underline{k}}^P\right]_{\underline{j}}$ is the input sparse coefficient of the bigger value $T_{k,j}(m)$. We define $\boldsymbol{A} \triangleq [\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_N]$ as the composite sparse representation where all $\boldsymbol{\alpha}_j(m)$, $\forall 1 \leqslant m \leqslant M$ are gathered. The all-in-focus image is then reconstructed as

$$\boldsymbol{I}_F = \boldsymbol{D}^F \boldsymbol{A}. \qquad (14)$$

The overall algorithm is summarized as Algorithm 1.

Algorithm 1 Image fusion from sparsity.

Input: the couple of training dictionaries $\boldsymbol{D}^F$ and $\boldsymbol{D}^B$; multi-focus source images $\{\boldsymbol{I}_k\}_{k=1}^n$.

1: Remove the mean intensity for source images.
2: **For** each 8×8 patch $\boldsymbol{i}_k$ from $\boldsymbol{I}_k$, starting from the upper-left corner with 1 pixel overlap,
· Solve the optimization problems (9), (10);
· Compute the averaging coefficients using (11);
· Generate the sparse coefficients $\boldsymbol{\alpha}_j(m)$ using (12), (13).
3: **End**
4: Reconstruct the all-in-focus image using (14).
5: Put the mean intensity into the all-in-focus image $\boldsymbol{I}_F$.

Output: the all-in-focus image $\boldsymbol{I}_F$

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3. SIMULATION RESULTS

This section illustrates the improvements for the primary parameters, and evaluate the proposed approach by visual comparisons and quantitative assessments. The assessments are based on two fusion performance metrics: (i) Q_{MI} [25], which measures how well the mutual information from source images is preserved in the fused image; (ii) $Q^{AB/F}$ [26], which evaluates how well the success of edge information transfers from source images to the fused image. All the experiments are performed on a PC running an Inter(R) Xeon(R) 3.40GHz CPU.

In our simulations, the training data consist of 50,000 8×8 focused patches which are randomly sampled from the database of 40 natural images and 50,000 8×8 blurred patches created by focused patches using Gaussian blur function. The two dictionaries are produced by the improved K-SVD algorithm, and initialized with samples from the training data. Considering the tradeoff between fusion quality and computation, we fix the dictionary size as 64×256, execute 6 multiple dictionary update cycles (DUC) and 30 iterations, and set the target cardinality as $T_0 = 6$. Experiments converge very quickly, and approximately achieve a $\times 2$ speedup compared to the standard coupled dictionary training [17].

We evaluate the proposed approach on the grayscale multifocus dataset which contain 10 pairs of grayscale images and 30 pairs of artifical images. Fig. 1 shows average fusion per-
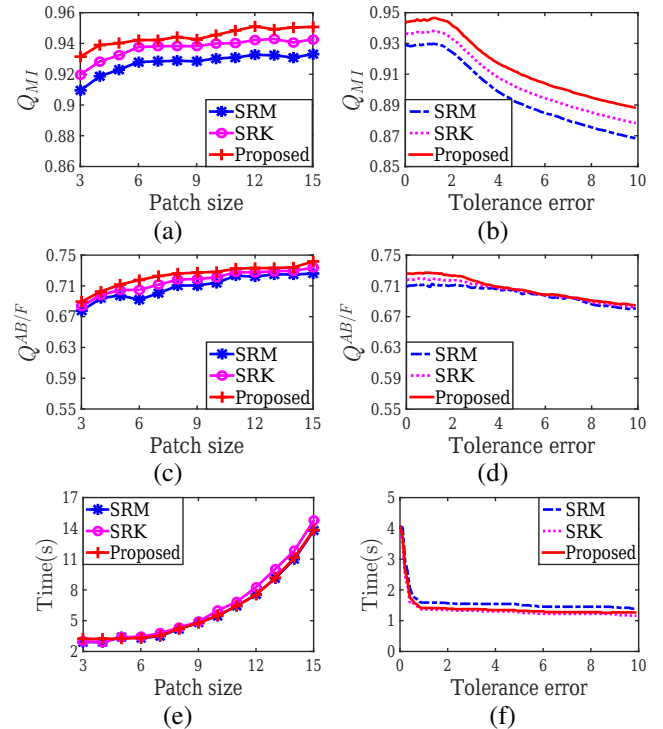

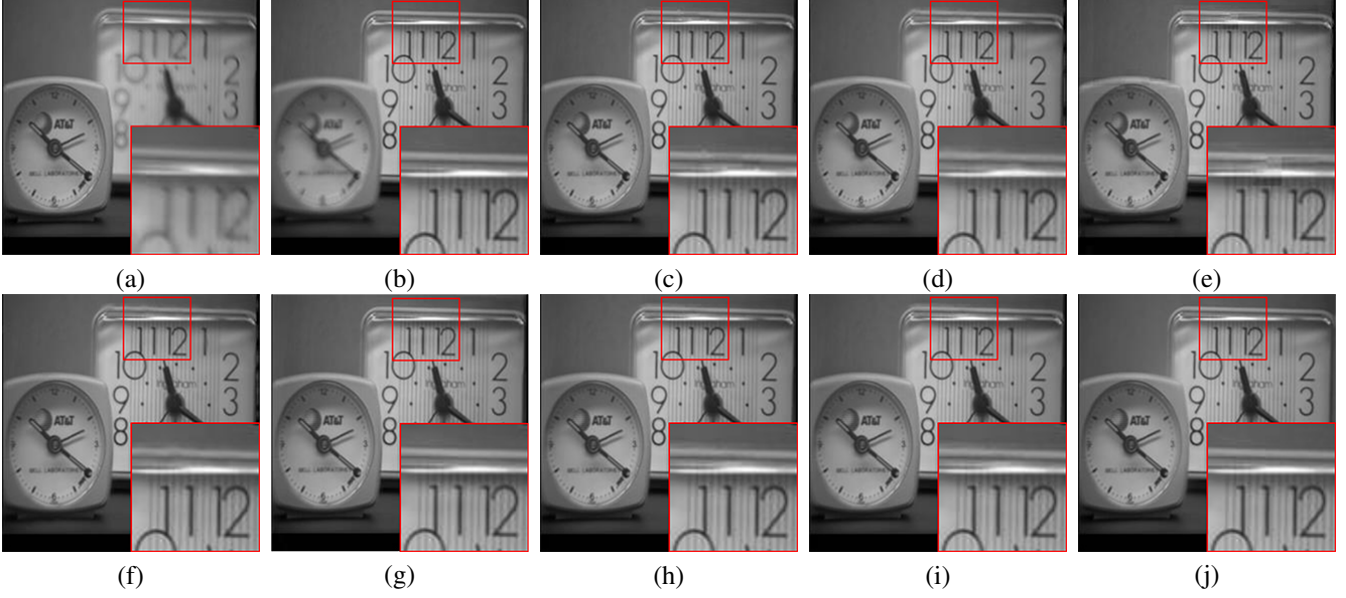**Fig. 1**. *Fusion performances versus patch size and tolerance error.*

Fig. 2. *Source images "Lab" and the fusion result comparisons. (a) The first source image with focus on the left. (b) The second source image with focus on the right. Fused images obtained by LP (c), MWG (d), DWT (e), NSCT (f), PCA (g), SRM (h), SRK (i) and the proposed method (j).*

formances versus different patch size and tolerance error ϵ. The comparisons adopt sparse representation "choose-max"-based (SRM) [15] and sparse representation K-SVD-based (SRK) approaches. The approaches utilize just one learned focus dictionary obtained by discrete cosine transform and K-SVD with the same parameters, respectively. As can be seen in Figs. 1(a) and (c), both Q_{MI} and $Q^{AB/F}$ slightly benefit from increasing patch size. When the patch size increases to 9, the running time begins to increase sharply (see Fig. 1(e)). Balancing computation time and fusion quality, we set the patch size to 8. Fig. 1(b) shows that the tolerance error slightly impacts on $Q^{AB/F}$. When ϵ is larger than 2, Q_{MI} is drastically decreasing. The simulations set $\epsilon = 1$. Additional experiments show that Q_{MI} and $Q^{AB/F}$ drastically decrease when number of overlapping pixels increases, so the overlapping size is set to 1.

To show the effectiveness of the proposed algorithm, we compare it with the existing algorithms, including LP [5], MWG [7], DWT [9], NSCT [10], PCA [1], SRM, and SRK approaches. The fusion results of the images "Lab" are shown in Fig. 2, including the magnified details in the lower right corners. We observe clearly that the LP method obviously lacks edge information (see Fig. 2(c)) and the DWT method results in blocking artifacts (see Fig. 2(e)). The fusion methods based on MWG (see Fig. 2(d)) and NSCT (see Fig. 2(f)) show circle blurring effect around strong boundaries. In Figs. 2(g), (h), and (i) some artificial distortions can be seen. Our approach provides the best visual appearance (see Fig. 2(j)). Tests with other images lead to similar results, and are thus omitted. For quantitative comparison, the evalu-

ations on grayscale multi-focus images are shown in Table 1. The values of Q_{MI} and $Q^{AB/F}$ range from 0 to 1, with 1 representing the ideal fusion. The bold values are the best results in the corresponding columns. As can be seen from Table 1, the proposed approach generally produces better quantitative results in terms of Q_{MI} and $Q^{AB/F}$. The value of Q_{MI} for the proposed approach is always the largest. That means that the proposed approach well preserves the mutual information from different source images. The value of $Q^{AB/F}$ demonstrates that our approach reduces the blocking artifacts and artificial distortions, and combines the significant edge information into the fused image.

Table 1. Objective evaluation of the image fusion

Measures	Methods							
	LP	MWG	DWT	NSCT	PCA	SRM	SPK	Ours
Q_{MI}	0.9083	0.9045	0.8991	0.9245	0.9381	0.9276	0.9386	**0.9432**
$Q^{AB/F}$	0.6879	0.7243	0.7013	0.7185	0.6620	0.7214	0.7326	**0.7451**

4. CONCLUSION

In this paper, we have proposed a novel multi-focus image fusion approach via jointly training the coupled dictionary. The proposed approach utilizes the couple of dictionaries from the focused and blurred categories of images to obtain multiple sparse representations, and exploits an effective and accurate fusion rule for estimating these representations. Experiments show that the proposed approach well preserves the edge and structural information of source images, and drastically reduces the blocking artifacts, circle blurring, and artifical distortions.

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